

Impact of VSWR on the Uncertainty Analysis of Harmonics For a Scope Calibrator Instrument

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Abstract

This discussion focuses on type A and B error analysis for a test station used to verify a high bandwidth oscilloscope calibrator. It is intended as a tutorial for existing and new calibration personnel to develop their uncertainty analysis for other high frequency systems. This presentation addresses, in detail, one of the major error contributors, voltage standing wave ratio (VSWR).

Introduction

Calibrating and verifying an oscilloscope calibrator requires a wide breadth of test instrumentation and switching capabilities. The specifications of a modern calibrator may include: 1) AC/DC voltage amplitude, 2) AC/DC current amplitude, 3) voltage measurement, 4) frequency/period, 5) leveled sine amplitude, 6) edge rise time, 7) aberration, 8) harmonics, 9) pulse width, 10) jitter, 11) skew, 12) triangle wave ramp linearity, 13) trigger amplitude, 14) resistance/capacitance, 15) overload, 16) external reference input, and 17) auxiliary input capability.

One of the difficult areas of verification occurs with the measurement of leveled sine harmonics. New scope calibrators output leveled sine signals that can exceed 2 GHz in frequency. At these higher frequencies, uncertainties associated with VSWR become an important contributor in the overall uncertainty analysis. This paper will focus on an uncertainty analysis of the leveled sine harmonic measurement.

Impact of Harmonics on Leveled Sine Amplitude

Harmonics contribute to the uncertainty of the leveled sine measurement by causing offset and distortion of the main or carrier signal. Chart 1 shows a typical leveled sine signal with exaggerated 2nd and 3rd harmonics (-20 dB lower in value). The 2nd harmonic signal causes an offset and the 3rd and other odd harmonics cause distortion of the signal. Measuring the harmonics of the leveled sine signal can give considerable insight into the quality of the signal being delivered to the scope.

In the extreme case, since the leveled sine signal is often used to verify the scope bandwidth, significant harmonics in the leveled sine signal can make it appear that the scope frequency response is less than specified. A lower 3dB amplitude may indicate that less signal is reaching the scope because the higher harmonics are being reflected or filtered by the scope input circuits. An issue that can complicate the calibrator harmonic measurement is the impedance mismatch between the signal source and load. At some frequency, the source output impedance and the load impedance stop looking like 50 ohms. When that occurs, a standing wave voltage is created.

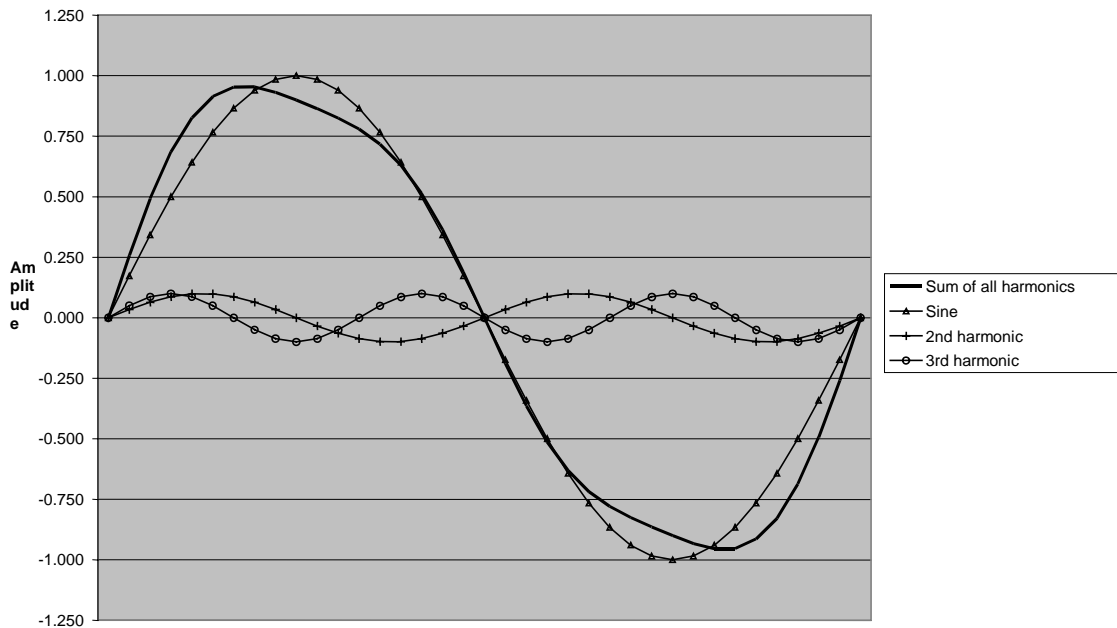


Chart 1: Combined impact of 2nd and 3rd harmonics on leveled sine amplitude

To summarize, 3rd harmonic is the most important when the peak to peak voltage is the quantity of interest. The 2nd harmonic adds a dc component that is usually not important. Higher order harmonics are generally are lower levels and do not have the impact as the 3rd harmonic.

Impact of VSWR

VSWR is a ratio of the reflected and incident voltage:

$$\text{VSWR} = \frac{(\text{V}_{\text{incident}} + \text{V}_{\text{reflected}})}{(\text{V}_{\text{incident}} - \text{V}_{\text{reflected}})}$$

It can also be thought of as the impedance mismatch between the elements in a system (generator, receiver, cabling, etc.). In most RF systems, VSWR measures the impedance mismatch to 50 ohms. In a perfect match, the VSWR would be = 1. In most calculation, reflection coefficients (Γ) are used instead of VSWR. While there is both a magnitude and a phase associated with reflection

coefficients, in the following discussions, only the magnitude of the reflection coefficients ρ , will be used. Their relationship is as follows:

$$\mathbf{VSWR = (1 + \rho) \div (1 - \rho)}.$$

The data in Chart 1, above, assumes that the signal generator is well matched to the impedance of the system load. Suppose that the output impedance of the source (Z_O) is not the same as the system (Z_L) to which it is attached. Due to the mismatch, a standing wave voltage will be generated as the signal reflects off of the load.

The maximum measurement uncertainty caused by VSWR mismatch can be calculated based on the reflection coefficients of the signal generator (Γ_G) and the system load (Γ_L). This will give us a worst case value. The reflection coefficient can be calculated from the VSWR and as shown in the following formulas:

$$\begin{aligned} \text{Magnitude of } \Gamma_G &= \frac{\text{VSWR}_G - 1}{\text{VSWR}_G + 1} \\ \text{Magnitude of } \Gamma_L &= \frac{\text{VSWR}_L - 1}{\text{VSWR}_L + 1} \end{aligned}$$

The resulting calculations are as follows:

$$\mathbf{Uncertainty\ limits\ (dB) = 20\log [1 +/- | \Gamma_G \times \Gamma_L |]}$$

$$\mathbf{Uncertainty\ Max\ (dB) = 20\log [1 + | \Gamma_G \times \Gamma_L |]\ \text{and}}$$

$$\mathbf{Uncertainty\ Min\ (dB) = 20\log [1 - | \Gamma_G \times \Gamma_L |].}$$

As an example, if the VSWR of a signal source at 2 GHz was < 1.5:1, and the VSWR of a spectrum analyzer to which it was attached was < 2.0:1, the following calculations would result.

$$|\Gamma_G| = \frac{1.5 - 1}{1.5 + 1} = \frac{0.5}{2.5} = 0.200$$

$$|\Gamma_L| = \frac{2.0 - 1}{2.0 + 1} = \frac{1.0}{3.0} = 0.333$$

$$\text{Power Uncertainty Max (dB) = } 20 \times \log [1 + | \Gamma_G \times \Gamma_L |] = +0.560 \text{ dB}$$

$$\text{Power Uncertainty Min (dB) = } 20 \times \log [1 - | \Gamma_G \times \Gamma_L |] = -0.599 \text{ dB}$$

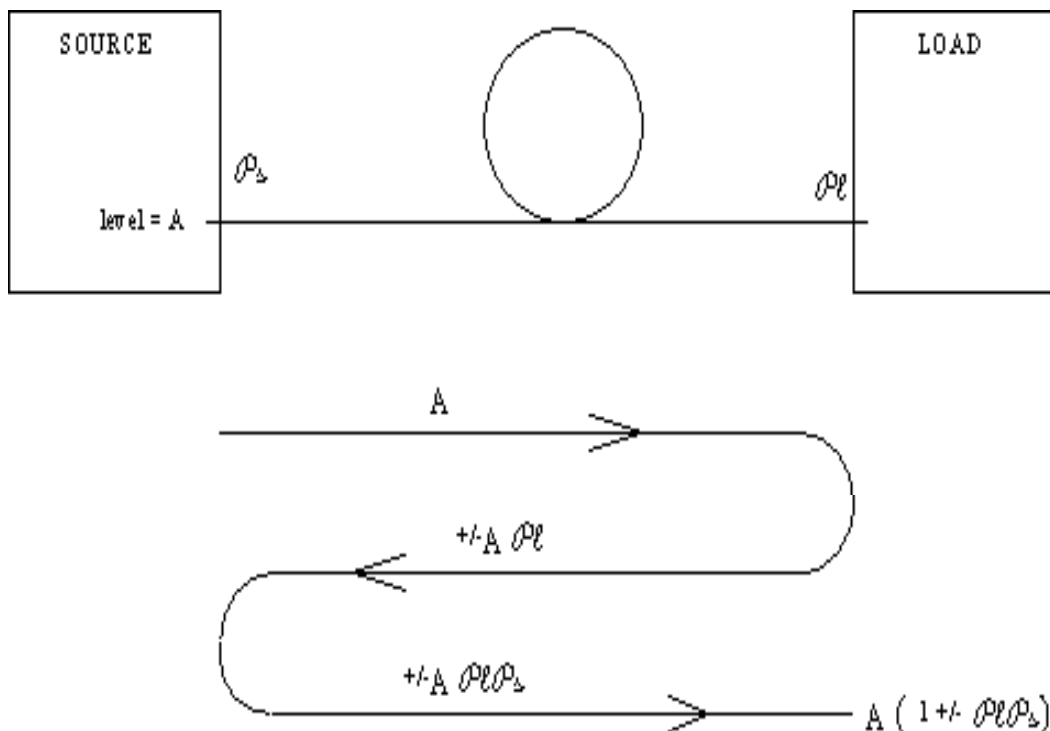


Figure 1. Illustration of Reflection Coefficient Properties (VSWR)

As a relative percentage¹, the following formulas apply:

$$\begin{aligned} \text{\%power mismatch uncertainty ratio} &= 100 \times [(1 \pm |\Gamma_G \times \Gamma_L|)^2 - 1] \\ &= +13.78\% \\ &= -12.89\% \end{aligned}$$

$$\begin{aligned} \text{\%voltage mismatch uncertainty ratio} &= 100 \times |\Gamma_G \times \Gamma_L| \\ &= \pm 6.67\% \end{aligned}$$

From our example, the maximum percent mismatch for voltage equals $\pm 6.67\%$. The mismatch can be a significant source of error in the uncertainty analysis and a method to reduce this error will be discussed later.

As a second illustrative example, Chart 1 shows the VSWR for a N_m to BNC_m cable that can be used in scope calibrator verification. VSWR measurements were made at the source end or BNC_m connector using a network analyzer. Note that the VSWR is frequency dependent. Frequency dependency means that the VSWR for the carrier signal may be different than for the harmonic signal. Generally, the VSWR is specified as a maximum value for the frequency range of a component. As an example, in Chart 2, the VSWR would be rated at less than 1.25:1 over the frequency range 500MHz to 2.1 GHz.

¹ See HP application note, AN 64-1A, page 44, for additional information.

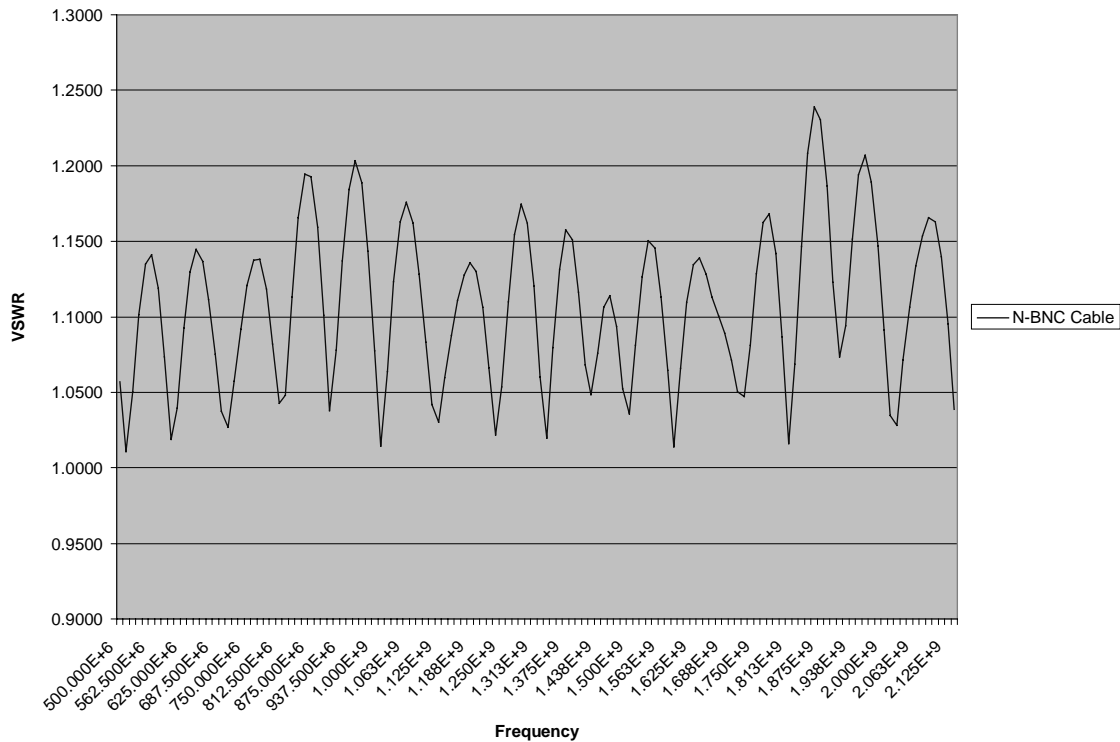


Chart 1: VSWR variation over frequency

The affects of VSWR are generally considered random in nature and must be added to the overall uncertainty analysis. At this point, a method to reduce the mismatch error will be discussed.

Reducing VSWR Mismatch Uncertainty

If we were to connect the calibrator directly to a spectrum analyzer, the uncertainty analysis would indicate that we might expect to see an error associated with the mismatched VSWRs.

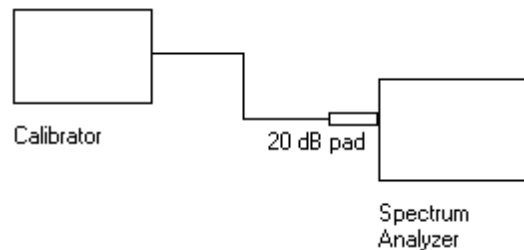


Figure 2. Connecting a 20 dB pad between a calibrator and Analyzer

From the VSWR example above where the calibrator had a VSWR of 1.5 and the analyzer had a VSWR of 2.0, we already saw a maximum uncertainty of > 6.67%. But by using an attenuator pad at the input to the spectrum analyzer, as in Figure 2, the VSWR mismatch can be reduced. The

addition of the attenuator, however, complicates the mismatch uncertainty formula. To illustrate this point, if we assume the attenuation is the same in either direction of attenuator, the ratio of reflected to sent voltage can be expressed as²:

$$\text{Attenuation} = \mathbf{V_{output}/V_{input}} = (10^{-att/20}) \div [1 +/- (\rho_G\rho_A + \rho_L\rho_A + 10^{-2 \times att/20} \times \rho_G\rho_L)]$$

ρ_A is the reflection coefficient of the attenuator in either direction

ρ_G is the reflection coefficient of the calibrator

ρ_L is the reflection coefficient of the analyzer

-att is the attenuator loss expressed as dB and we assume loss is the same either way through the attenuator.

The voltage uncertainty of this equation would be the denominator term as follows:

$$\mathbf{u(att)} = \rho_G\rho_A + \rho_L\rho_A + 10^{-2 \times att/20} \rho_G\rho_L.$$

If we assume that the attenuator has a gain of -20 dB, and vary its VSWR values, a dramatic impact on the overall uncertainty can be demonstrated. If we use the same VSWRs for the calibrator and load as previously, Table 1 results. For the attenuator, values of VSWR below 1.28 reduce the overall uncertainty while values above 1.28 increase the uncertainty.

	Calibrator VSWR	Analyzer VSWR	Attenuator VSWR	Attenuator (dB)	Actual Attenuation	(+/-) Voltage Uncertainty due to mismatches
No Attenuator	1.5	2	1	0	0.938	6.67%
Attenuator VSWR 1.3	1.5	2	1.3	20	0.093	7.02%
Attenuator VSWR 1.282	1.5	2	1.282	20	0.094	6.66%
Attenuator VSWR 1.2	1.5	2	1.2	20	0.095	4.92%
Attenuator VSWR 1.15	1.5	2	1.15	20	0.096	3.79%
Attenuator VSWR 1.1	1.5	2	1.1	20	0.097	2.61%
Attenuator VSWR 1.075	1.5	2	1.075	20	0.098	1.99%
Attenuator VSWR 1.05	1.5	2	1.05	20	0.099	1.37%
Attenuator VSWR 1.015	1.5	2	1.015	20	0.100	0.46%
Attenuator VSWR 1.15	1.5	2	1.15	10	0.303	4.39%
Attenuator VSWR 1.15	1.5	2	1.15	3	0.661	7.06%

Table 1: Impact of Attenuator VSWR on Mismatch

The important point is that, to improve the overall mismatch, an attenuator pad can be used. However, the attenuator’s VSWR must be less than the VSWR of the connecting devices. By using an attenuator with a VSWR of 1.075, the mismatch error for the analyzer to calibrator is reduced from 6.7% to 2%. Note also that the value of the attenuator is important. If the attenuator VSWR remains the same but its attenuation is reduced to 10dB or 3dB, the voltage uncertainty also changes. Thus, for a given application, the VSWR and attenuation value must be considered.

² “SINGLAS Technical Guide - Guidelines on the Evaluation and Expression of Uncertainty of Measurement Uncertainty: Example 5 – Harmonic Measurement”, Singapore Institute of Standards and Industrial Research, July1995.

One other point before we leave this section. The probability distribution for the VSWR mismatch follows a “U” shape³. This means that the uncertainty of the mismatch is more likely to occur near the ends of the range. As a result, the square root of 2 should be used as the normalizing 1-sigma factor in the uncertainty analysis. For the case where the VSWR of the attenuator is 1.075, the maximum uncertainty ratio would be = 0.02/sqrt(2) = 0.014.

Harmonic Measurement Uncertainty Analysis

We can combine the affects of VSWR on harmonics in a real measurement example. Harmonic measurement is a function of carrier power, harmonic power, mismatch path loss between calibrator and analyzer (including effects of VSWR), variation in path loss (excluding affects of VSWR), and uncertainty introduced by the spectrum analyzer. Combining these terms produces the following harmonic measurement equation⁴:

$$\text{Harmonic ratio} = \frac{\text{Power}_{\text{Harmonic}}/\text{Pathloss}_{\text{Harmonic}} \times (1 + E_{\text{flatness}} + SA_{\text{unc}})}{\text{Power}_{\text{Carrier}}/\text{Pathloss}_{\text{Carrier}}}$$

- Power_{Harmonic} is the measured level of the harmonic level (mW).
- Power_{Carrier} is the measured level of the carrier signal (mW).
- Pathloss_{Harmonic} is the path loss for the harmonic signal between the scope calibrator output and the spectrum analyzer including mismatch.
- Pathloss_{Carrier} is the path loss for the carrier signal between the scope calibrator output and the spectrum analyzer including mismatch.
- E_{flatness} is the variation in path loss for the attenuators and adapters (not including VSWR) when measuring the harmonic and carrier signals.
- SA_{unc} is the uncertainty introduced by the spectrum analyzer.

Taking the partial derivative of the Harmonic equation with respect to carrier power, harmonic power, mismatch path loss between calibrator and analyzer for the carrier (including effects of VSWR), mismatch path loss between calibrator and analyzer for the harmonics (including effects of VSWR), variation in path loss (excluding affects of VSWR), and uncertainty introduced by the spectrum analyzer produces the following uncertainty equation:

$$u(\text{Har}) = \sqrt{\frac{-\text{Power}_{\text{Harmonic}}^2}{\text{Power}_{\text{Carrier}}^2} \times \left[\frac{u(\text{Pathloss}_{\text{Carrier}})^2}{(\text{Pathloss}_{\text{Carrier}})^2} + \frac{u(\text{Pathloss}_{\text{Harmonic}})^2}{(\text{Pathloss}_{\text{Harmonic}})^2} + u^2(E_{\text{flatness}}) + u^2(SA_{\text{unc}}) \right]}$$

³ I.A. Harris and F.L. Warner, “Re-examination of Mismatch Uncertainty when Measuring Microwave power and Attenuation”, Proceeding of the British IEE, Vol. 128, pp. 35-41, Feb. 1981.

⁴ “SINGLAS Technical Guide - Guidelines on the Evaluation and Expression of Uncertainty of Measurement Uncertainty: Example 5 – Harmonic Measurement”, Singapore Institute of Standards and Industrial Research, July 1995.

For this application, we assume the signal path of the carrier and the harmonics to be the same and so the path loss will also be the same. Reducing terms produces the following uncertainty equation:

$$u(\text{Har}) = \frac{\text{Power}_{\text{Harmonic}}}{\text{Power}_{\text{Carrier}}} \times \sqrt{ \left[2 \times \frac{u(\text{Pathloss}_{\text{Harmonic}})^2}{(\text{Pathloss}_{\text{Harmonic}})^2} + u^2(\text{E}_{\text{flatness}}) + u^2(\text{SA}_{\text{unc}}) \right]}$$

Type A Uncertainty

As part of the harmonic test development strategy, multiple calibrators must be tested and statistics for the harmonic measurements determined. A partial list of statistics for 38 calibrators is shown in Table 3 for a few selected frequencies at 0.35Vpp. Harmonics are measured relative to a carrier or main frequency and are often expressed in dB down from the carrier or sometimes as dBc.

Type A Harmonic Measurements (max)	Signal Voltage	Freq. (Hz)	Harmonic Spec (dB)	38 Measurements Average Har (V)	1-σ Unc(Har) (V)
2ndharmonic	0.35	1.475E+9	-33	0.017973	0.000224
3rd+harmonic	0.35	1.475E+9	-38	0.004252	0.000115
2ndharmonic	0.35	1.525E+9	-33	0.004406	0.000229
3rd+harmonic	0.35	1.525E+9	-38	0.005787	0.000127
2ndharmonic	0.35	1.575E+9	-33	0.015428	0.000312
3rd+harmonic	0.35	1.575E+9	-38	0.003993	0.000114

Table 3: Results of 38 measurements of Harmonics

Average harmonic amplitudes and the standard deviation were derived from the data and expressed in volts. It is recommended to always avoid working in dB for uncertainty analysis unless the result you are seeking is also in dB.

The values listed represent the largest values for the standard deviations over the frequency range. They represent the amount of measurement error that can be expected in the system to 1 sigma level. From this data, the max uncertainty voltage would be 312 uV.

The calibrator also has its own contribution to VSWR mismatch. The VSWR of the calibrator can be measured using a spectrum analyzer and special mismatch loads. Results of these measurements created the following chart. Note that the maximum voltage output is also the range where a calibrator has no attenuation. This represents the worst case mismatch. In the chart, the VSWR of 1.5:1 is used for the uncertainty analysis.

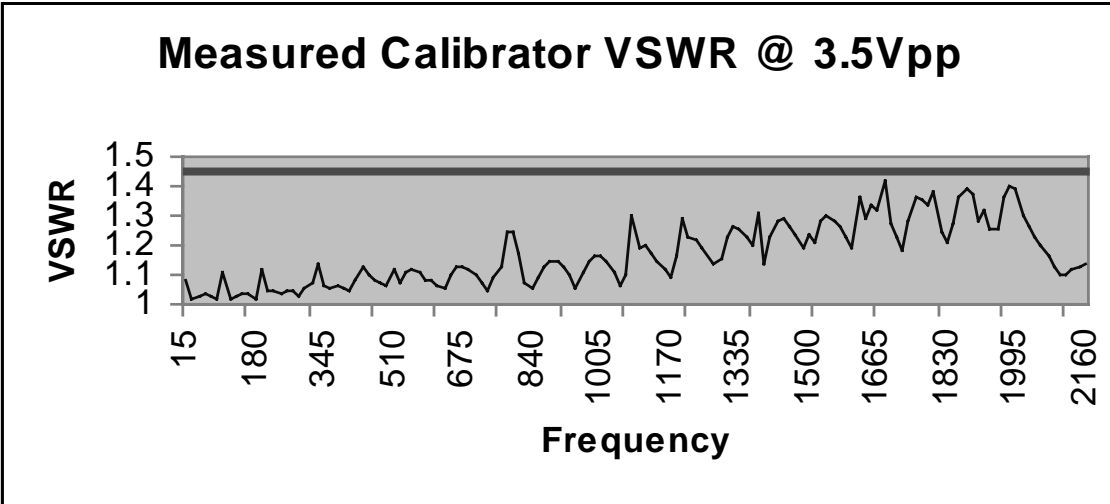


Chart 2: Measured Calibrator VSWR frequency

Path loss variation is another element of the Type “A” uncertainty. In measuring the loss of a number of cables from several manufacturing date codes, the following cable to cable loss uncertainty chart was developed. Note that the maximum loss deviation is 0.05 dB (0.6%).

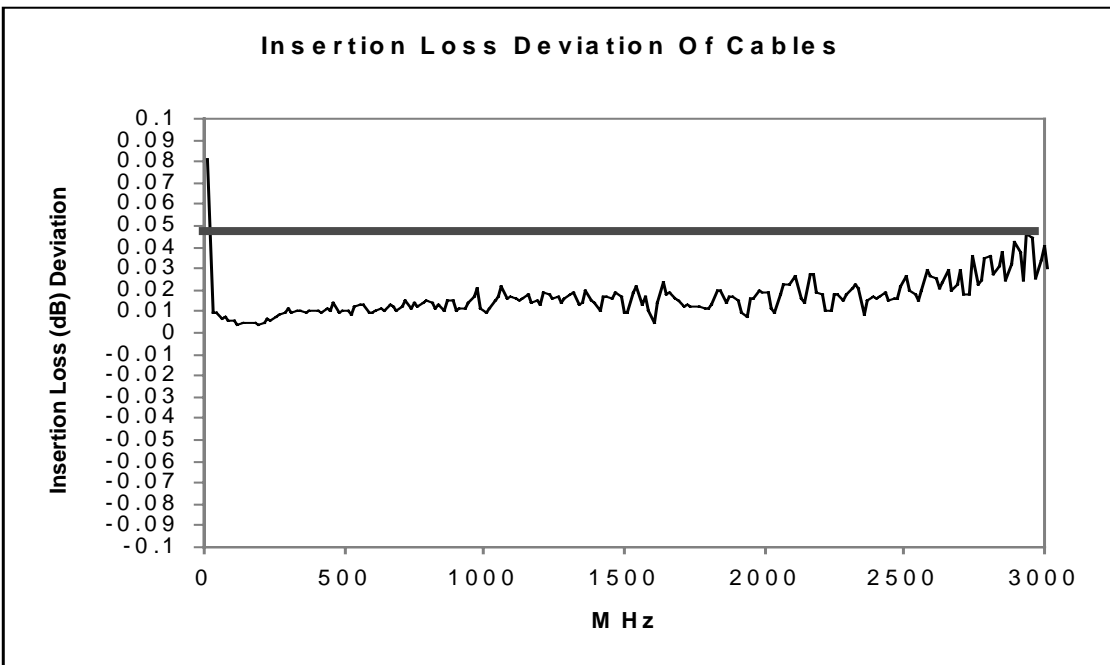


Chart 3: Measured Cable to Cable Loss Variation

Type B Uncertainty Sources

Several of the measurement uncertainties were taken from the data sheets. The spectrum analyzer and attenuator flatness were examples. In some cases, the type “B” uncertainty had to be mixed with type “A” uncertainty. The predominate uncertainty source determined how the uncertainty was handled in the summary analysis – as a type “A” or a type “B”.

The spectrum analyzer had two important error sources. The first source was its VSWR. For this analysis, a VSWR of 2.0:1 was used. Noise, gain, temperature and non-linearity errors were combined into an uncertainty of +/- 1 dB (12%).

Path loss due to VSWR mismatches between the spectrum analyzer and the calibrator were calculated previously as 6.67%. When a 20 dB attenuator with the VSWR < 1.075 was inserted between the spectrum analyzer and the calibrator, the mismatch error was reduced to 2%. This was a case where the type “A” uncertainty of the calibrator VSWR measurement was combined with type “B” uncertainties.

The path loss flatness variation of the cable and the attenuator had to be considered. From the data sheet, the maximum uncertainty of the attenuator was < 0.5 dB (5.9%). We saw earlier that the cable loss uncertainty was measured to be 0.05 dB (0.6%). Combining the uncertainties in an RSS fashion generated an uncertainty of 5.93%.

Summary Analysis

Table 3 lists uncertainties and values that were derived in a Scope Calibrator analysis. The uncertainty voltages for the type “B” terms were generated by multiplying the uncertainty ratios by the harmonic voltage specification. As an example, for spectrum analyzer uncertainty, at 0.35Vpp, the harmonic voltage would be $0.35 * 10^{(-33/20)} = 0.0078\text{Vpp}$. The uncertainty would be 12% times that value or 940 uV. To convert that value into a standard uncertainty, it was divided by $\sqrt{3}$.

Mismatch errors were bundled into the “Attenuator with mismatch” mismatch ratio (2.0%) as demonstrated in Table 1 above. That ratio was multiplied by the harmonic voltage specification to derive the harmonic uncertainty voltage for mismatch (157 uV). To derive the 1-sigma coverage for mismatch uncertainty, the term was divided by $\sqrt{2}$.

The flatness uncertainty, $U(E_{\text{flatness}})$, was derived from the uncertainty associated with the flatness of the attenuator (derived from the specification sheet) and the differences in flatness through the cable (specification sheet and verified by test) for the carrier and its harmonics over the frequency range for a total uncertainty of 464 uV. This value was divided by the $\sqrt{3}$ to put it into a standard deviation format.

All of the type “B” terms were RSS'd together. The type “A” term was considered to already be in a 1-sigma form. The test uncertainty ratio (TUR) was calculated as:

$$= V_{\text{out}} \times [10^{(\text{har_spec}(\text{dB})/20)}] \div E_{\text{u}}(\text{v})$$

$$= V(\text{har})/Eu(v) = 0.0078V/0.00138V = 5.7$$

Source of Uncert.	Type	Ui	Unc Value	Probability Distribution	Coverage Factor	Standard Uncert.	Degrees of Freedom
Spectrum Analyzer* (V)	B	$U(\text{TDS}_{\text{error}})$	$0.12 \times 0.0078 = 940 \text{ uV}$	rectangular	$\sqrt{3}$	0.000542	infinite
Attenuator incl. mismatch VSWR 1.075 (V)	B	$U(\text{Att}_A)$	$0.02 \times 0.0078 = 157 \text{ uV}$	U-shape	$\sqrt{2}$	0.000111	infinite
Path Loss Variation (V)	B	$U(E_{\text{flatness}})$	$0.0593 \times 0.0078 = 464 \text{ uV}$	rectangular	$\sqrt{3}$	0.000268	infinite
Repeatability (38 readings)	A	$U_A(\text{Har})$		normal	1	0.000312	37
Total Type A Uncertainty			0.000312				
Total Type B Uncertainty		$\sqrt{U(\text{Att}_A)^2 + 2 \times U(E_{\text{flatness}})^2 + U_A(\text{Har})^2}$	0.000616				
Combined Uncertainty		$\text{SQRT}(A^2+B^2)$	0.000690				
Expanded Uncertainty	Eu(v)	2 x Combined	0.00138				
TUR	= $(V_{\text{out}} * 10^{(\text{har dB}/20)})/\text{Expanded Uncertainty}$					5.67:1	

*uncertainties included: temperature, interpolation, gain, noise, and integral non-linearity but does not include VSWR.

Table 3: Uncertainty Summary for Harmonics

Summary

This discussion focused on type A and B error analysis for a test station used to verify a high bandwidth oscilloscope calibrator. One of the difficult areas of verification occurs with the measurement of leveled sine harmonics. New scope calibrators output RF leveled sine where the uncertainties associated with VSWR become an important contributor to the overall uncertainty analysis. A method was shown that could be used to reduce the affects of VSWR mismatch using an attenuator. A method was also shown how to sum the various uncertainty parameters into a unified TUR value. Millennium Ford